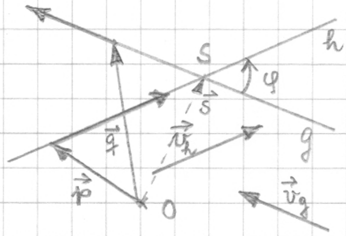
SCHNITTMENGEN – WINKEL

ma – 3

1) GERADE – GERADE



Bedingung: (1) $\vec{s} = \vec{p} + k_s \cdot \vec{v_h} = \vec{q} + c_s \cdot \vec{v_g}$

(2) $\varphi = \arccos\left(\frac{\vec{v_h}\cdot\vec{v_g}}{v_h \cdot v_g}\right)$

Bsp.: $g: \vec{x} = \begin{pmatrix}9\\5\\0\end{pmatrix} + k\cdot\begin{pmatrix}2\\1\\-2\end{pmatrix}$; $h: \vec{x} = \begin{pmatrix}6\\3\\1\end{pmatrix} + c\cdot\begin{pmatrix}1\\0\\-3\end{pmatrix}$

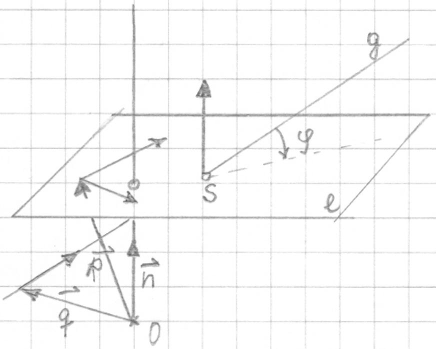
$$\Rightarrow \left.\begin{array}{ll} ① & 9+2k_s = 6+c_s \\ ② & 5+k_s = 3 \\ ③ & -2k_s = 1-3c_s \end{array}\right\} \Longleftrightarrow \left\{\begin{array}{l} k_s = -2 \\ c_s = -1 \end{array}\right\} \quad 9+2\cdot(-2) = 6-1 \text{ wahr!}$$

$S(5|3|4)$

$\varphi = \arccos\left(\frac{8}{3\cdot\sqrt{10}}\right) \Rightarrow \varphi \approx 32{,}5°$

Beachte: Auch windschiefe Geraden schließen miteinander einen Winkel ein!

2) GERADE – EBENE



Bedingung: (1) $(\vec{s} = \vec{q} + c_s \cdot v_g) \wedge (\vec{s}\cdot\vec{n} = \vec{p}\cdot\vec{n})$

(2) $\varphi = \arcsin\left(\frac{\vec{v_g}\cdot\vec{n}}{v_g \cdot n}\right)$

Bsp: $e: \quad 2x - 3y + z = 4$; $g: \vec{x} = \begin{pmatrix}9\\5\\0\end{pmatrix} + k\cdot\begin{pmatrix}2\\1\\-2\end{pmatrix}$

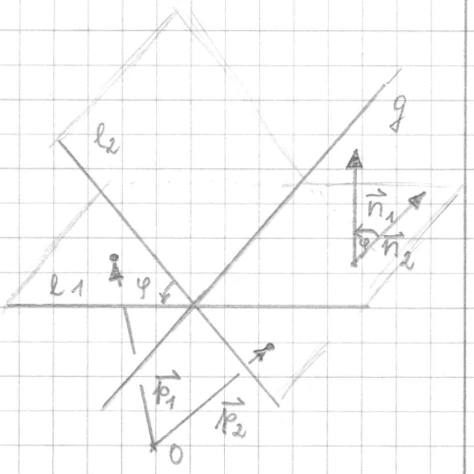
$\Rightarrow 2\cdot(9+2k_s) - 3\cdot(5+k_s) - 2k_s = 4 \Rightarrow k_s = -1$

$S(7|4|2)$

$\varphi = \arcsin\left(\frac{-1}{\sqrt{14}\cdot 3}\right) \approx -5{,}1° \quad (+5{,}1°)$

$\cos(90° - \varphi) = \sin\varphi$

3) EBENE – EBENE



Bedingung: ① $(\vec{s}\vec{n_1} = \vec{p_1}\vec{n_1}) \wedge (\vec{s}\vec{n_2} = \vec{p_2}\vec{n_2})$

② $\varphi = \arccos\left(\frac{\vec{n_1}\cdot\vec{n_2}}{n_1\cdot n_2}\right)$

Bsp: $$\left.\begin{array}{ll} e_1: & 2x - 3y + z = 4 \\ e_2: & -x + 2y + z = 1 \end{array}\right\} \Rightarrow \quad y + 3z = 6 \quad (① + 2\cdot②)$$

Wahl: $z = t \in \mathbb{R} \Rightarrow y = 6 - 3t \Rightarrow x = t - 1 + 2(6 - 3t)$

$g: \vec{x} = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}11-5t\\6-3t\\t\end{pmatrix} = \begin{pmatrix}11\\6\\0\end{pmatrix} + t\cdot\begin{pmatrix}-5\\-3\\1\end{pmatrix}$

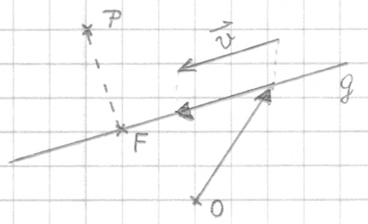
$\varphi = \arccos\left(\frac{-7}{\sqrt{14}\cdot\sqrt{6}}\right) \approx 139{,}8° \quad (40{,}2°)$

Beispielaufgaben der Analytischen Geometrie

ABSTÄNDE

ma-3

1) PUNKT – GERADE



<u>Bedingung:</u> $\overrightarrow{PF} \cdot \vec{v} = 0$

Bsp: $P(2|-1|3)$; $g: \vec{x} = \begin{pmatrix}1\\1\\1\end{pmatrix} + k\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix}$

$\Rightarrow \overrightarrow{PF} = \begin{pmatrix}1-k_f\\1+2k_f\\1+2k_f\end{pmatrix} - \begin{pmatrix}2\\-1\\3\end{pmatrix} = \begin{pmatrix}-1-k_f\\2+2k_f\\-2+2k_f\end{pmatrix}$

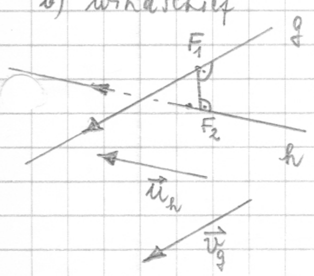
$\Rightarrow (-1)\cdot(-1-k_f) + 2\cdot(2+k_f\cdot 2) + 2\cdot(-2+2k_f) = 0$

$\Rightarrow 1 + 9k_f = 0 \Rightarrow k_f = -\frac{1}{9} \Rightarrow \overrightarrow{PF} = \frac{1}{9}\begin{pmatrix}-8\\16\\-20\end{pmatrix}$

$\Rightarrow d = |\overrightarrow{PF}| = \frac{4}{9}\cdot\sqrt{45} = \frac{4}{3}\cdot\sqrt{5}$

2) GERADE – GERADE

a) parallel, dann 1)
b) windschief



<u>Bedingung:</u> ① $\overrightarrow{F_1F_2} \cdot \vec{u} = 0$ und ② $\overrightarrow{F_1F_2} \cdot \vec{v} = 0$

Bsp: $g: \vec{x} = \begin{pmatrix}0\\-2\\0\end{pmatrix} + k\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix}$; $h: \vec{x} = \begin{pmatrix}0\\4\\0\end{pmatrix} + c\cdot\begin{pmatrix}-2\\-2\\1\end{pmatrix}$

$\Rightarrow \overrightarrow{F_1F_2} = \begin{pmatrix}0-2c_f\\4-2c_f\\0+c_f\end{pmatrix} - \begin{pmatrix}0-k_f\\-2+2k_f\\0+2k_f\end{pmatrix} = \begin{pmatrix}k_f-2c_f\\6-2c_f-2k_f\\c_f-2k_f\end{pmatrix}$

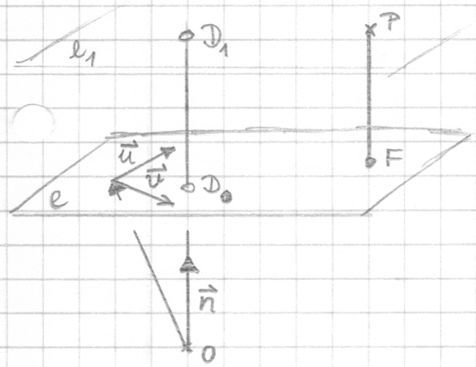
$\Rightarrow$ ① $(-2)\cdot(k_f-2c_f) + (-2)\cdot(6-2c_f-2k_f) + (c_f-2k_f) = 0$

$\wedge$ ② $(-1)\cdot(k_f-2c_f) + 2\cdot(6-2c_f-2k_f) + 2\cdot(c_f-2k_f) = 0$

$\Rightarrow$ ① $9c_f = 12$ $\wedge$ ② $9k_f = 12$

$\Rightarrow \overrightarrow{F_1F_2} = \frac{1}{3}\begin{pmatrix}-4\\2\\-4\end{pmatrix} \Rightarrow d = |\overrightarrow{F_1F_2}| = 2$

3) PUNKT – EBENE



<u>Bedingung:</u> $|\overrightarrow{PF}| = \overline{D_1O} - \overline{D_0O}$ und $e_1 \parallel e$

e_1 ist Hilfsebene mit $P \in e_1$!

Bsp.: $P(2|-1|3)$; $e: \vec{x} = \begin{pmatrix}1\\1\\1\end{pmatrix} + k\cdot\begin{pmatrix}-1\\2\\2\end{pmatrix} + c\begin{pmatrix}-2\\-2\\1\end{pmatrix}$

$$\left.\begin{array}{l}① \; -n_1 + 2n_2 + 2n_3 = 0\\ \wedge \; ③ \; -2n_1 - 2n_2 + n_3 = 0\end{array}\right\} \Leftrightarrow \left\{\begin{array}{l}-n_1 + 2n_2 + 2n_3 = 0\\ -3n_1 + 3n_3 = 0\end{array}\right.$$

$\Rightarrow \vec{n} := \begin{pmatrix}2\\-1\\2\end{pmatrix}$

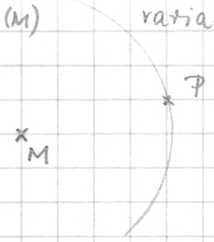
$\Rightarrow e: \begin{pmatrix}2\\-1\\2\end{pmatrix}\cdot\vec{x} = 3$; $e_1: \begin{pmatrix}2\\-1\\2\end{pmatrix}\cdot\vec{x} = 11$

HNF: $e: \frac{1}{3}\begin{pmatrix}2\\-1\\2\end{pmatrix}\cdot\vec{x} = 1$; $e_1: \frac{1}{3}\begin{pmatrix}2\\-1\\2\end{pmatrix}\cdot\vec{x} = \frac{11}{3}$

$\Rightarrow |\overrightarrow{PF}| = \frac{8}{3}$

4) PUNKT – PUNKT

fest (M) variabel ($\vec{X}$)



<u>Bedingung:</u> $|\overrightarrow{XM}| = \text{konst} = |\overrightarrow{PM}|$ (Kugel)

Bsp: $\left(\vec{x} - \begin{pmatrix}-1\\4\\2\end{pmatrix}\right)^2 = \left(\begin{pmatrix}-1\\4\\2\end{pmatrix} - \begin{pmatrix}3\\2\\-2\end{pmatrix}\right)^2 = 36$

$M:(-1|4|2)$; $P(3|2|-2)$